A generalized Product of Experts model for non-linear utility specifications in travel behaviour models

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INTRODUCTION

Conventional approaches for discrete choice modelling assume that behaviour of the decision maker are derived from random utility maximization (RUM), which have been the standard practice for decades *(1, 2)*. They provide relatively realistic forecasts for mean and variances of various socio-economic variables in discrete choice applications *(3)*. The RUM model assumes a (dis)utility – a linear sum of attributes multiplied with a weight parameter – which determines the choice preference for the individual. This has been the most popular method of estimating travel demand due to its closed-form analytical tractability.

In recent years, there has been new branches of discrete choice modelling explored in literature: non-linear econometric behaviour models, Bayesian decision models and artificial neural networks *(4–6)*. These processes are still not well understood and remain underutilized in travel behaviour modelling. They cannot be easily defined by a simple linear mathematical formulation or quantified by static attributes to reflect variations that occur in the utility component. Decision heuristics have been defined for approximating these behaviour processes *(7, 8)*. Some notable examples include extreme aversion, similarity effect and compromise, incorporating the risk aversion and compromise effects into the RUM utility function *(9)*. They were designed to be an alternative solution to linear RUM but have not gained traction until recently due to their non-linear nature and difficulty in model inference.

This study proposes an analytical information maximization approach for a generalized multinomial discrete choice model where the probability distribution is a function of expected utility and entropy by using methods developed in information theory. Such formulation allows the modeller to define a simple, yet deterministic function for the preference behaviour through a multiplicative choice process.

We exploit the ideas from the Product of Experts (PoE) framework, a generic method for estimating high dimensional probability distributions *(10)*. The PoE framework provides a structured way of understanding non-linear models and what makes them practical and useful for complex choice problems. Inference is possible through Naïve Bayes assumption that assumes each dimension contribute independently to the choice probability.

METHODOLOGY

The PoE can be extended to incorporate various forms of heterogeneity by using Bayesian networks to model non-linearity in choice models. To date, only few studies have implemented the Bayesian approach e.g. *(11)*, however it has a strong potential to be used in so-called high-dimensional *Big Data* modelling and ubiquitous data services.

Consider a decision maker who has to select a single alternative among a discrete set of alternatives $Y = (y_1, y_2, ..., y_j)$. We can express this as a distribution by assigning the probability space $P(Y) = (p_1, p_2, ..., p_j)$, with $p_j = p(y_j)$. Given some unknown set of internal latent choice process $X = (x_1, x_2, ..., x_h)$ (i.e. 'experts'), the probability of the decision maker conditioned on the priors $P(Y|X)$ can be computed as a normalized product of these experts:

$$P(Y|x_1, x_2, ..., x_h) = \frac{1}{Z}\prod_{i=1}^{h} p(x_i|Y)f(Y), \tag{1}$$

where $Z = \sum_Y \prod_{i=1}^{h} p(x_i|Y)f(Y) = 1$ is the normalising partition function. By the Bayes rule, it is often denoted as:

$$P(Y|X) \propto P(X|Y)P(Y). \tag{2}$$

In contrast to a mixture model, where individual experts are combined additively and the weights of the mixture satisfy $\sum_i \omega_i = 1$:

$$P(Y|x_1, x_2, ..., x_h) = \sum_{i=1}^{h} \omega_i p(x_i|Y). \tag{3}$$

Under the PoE framework, the choice probability can alternatively be estimated through maximizing the information entropy. One of the interesting aspects of this information-theoretic approach is that we can apply two measures that describe the generalized behaviour of both action and perception by casting behaviour into (a) utility maximization and (b) uncertainty minimization known as the free energy principle *(12)*. It is assumed that given enough time, a decision maker will naturally converge into an optimal state of the environment and develop an internal state that learns to avoid uncertainties about choice preferences. Given enough (or lack of) explanatory features would mean that the decision maker can build a perception about the environment and adapts his or her behaviour based on the internal states. In the case of a classical RUM, this notion is simplified or ignored, however there are more sophisticated models (e.g. Mixed Logit, Latent Variable Models) that incorporate this uncertainty through the Kullback-Leibler (KL) divergence.

we can consider the maximum likelihood estimator (MLE) as the sum of entropy and the KL divergence. The KL divergence represents a 'distance metric' between two distributions $p(y_j)$ and $p_{\hat{\theta}}(y_j)$ or how (dis)similar are the distributions. This reformulation allows for two different ways of maximizing a given policy.

FINDINGS

A comprehensive case study was conducted to show the uptake in this new mode given some hypothetical attributes on a stated preference survey (Train Hotel). The survey was conducted among commuters from the Northeastern region of the United States and Quebec, Canada. An illustration of the routes and destinations between the two countries is shown in Figure 1. Respondents were asked to indicate their preferred option of travel. There are a total of 596 respondents where each person performed the survey 3 times with different alternative specific values for a total of 1788 responses. The survey design, question format and further details can be found in *(13)*.

We developed a dataset on mode choice of a new overnight commuter train compared to existing conventional travel modes. A stated preference (SP) survey is performed on regular and occasional commuters travelling along the Northeastern rail corridor. The survey analyses the feasibility of implementing the new travel mode (Train Hotel against existing options (private car, rental car, commuter train, plane, bus).

Train Hotel dataset is used in our experiment to determine the optimal likelihood for each

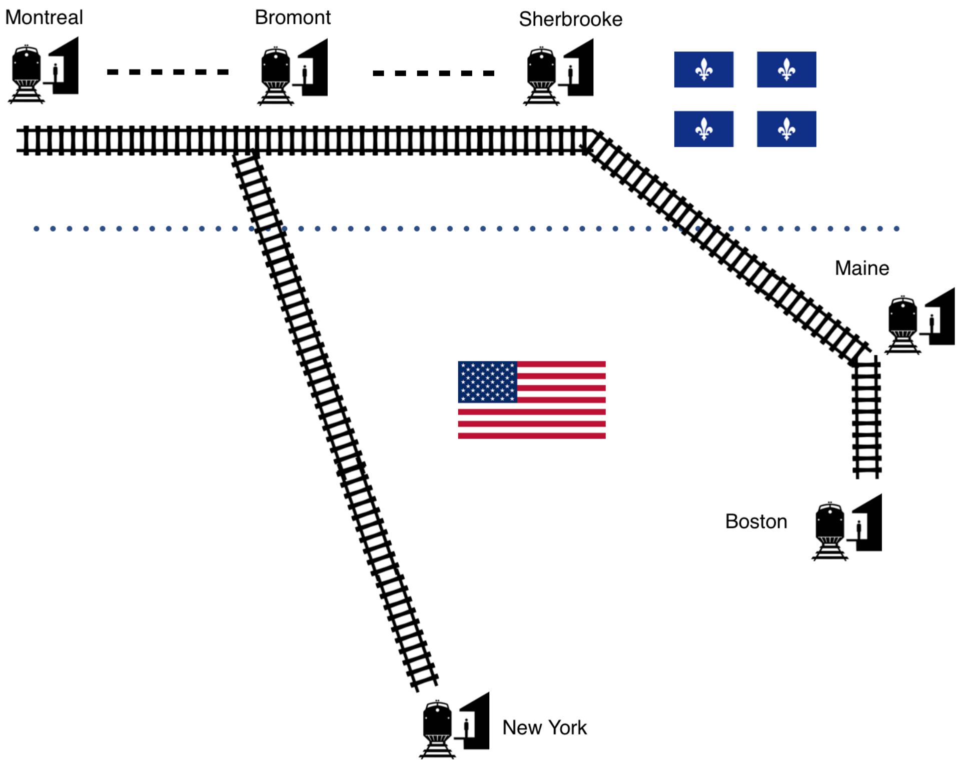

FIGURE 1 Map of the Northeastern rail corridor and service destinations.

model. We estimated model parameters for the alternative specific attributes: cost, travel time and reliability. The models were estimated on Python BIOGEME *(14)*. We compared 5 different model formulations: (a) Utility only (standard MNL) model, (b) Mixed Logit model, (c) classical RRM model (RRM2010), (d) PoE model and (e) entropy only model. In our PoE model, it may be possible to estimate individual attribute parameters in the expected utility, independent from entropy. Consistent with our expectations, the beta values estimated in the RRM and PoE formulations showed larger effects of cost and smaller effects of travel time on individual preference. The result shows that the PoE model obtained the highest performance in terms of log likelihood. By comparing the random effects of the Mixed Logit model, we observed that even though the Mixed Logit has the highest log likelihood, the AIC and BIC values indicate that it is not as efficient as the other model specifications. For the PoE model, it was estimated that cost (-0.266) and travel time (-0.035) has a larger effect on mode choice than RRM, (-0.186) and (-0.0232) respectively. The statistical significance of the parameters in the PoE model are greater than other RRM and MNL models, indicating that the PoE parameters are more accurate in representation. Another interesting finding is the entropy only and RRM2010 model are *exactly* the same. This surprising result provides an empirical and analytical proof that the estimation of a RRM2010 model specification does not always satisfy the utility maximization axiom.

From these results, we can determine that the classical RRM formulation does not optimize for *maximum utility*. While in fact, if the utility is reformulated, in the case of the PoE model, it may improve model accuracy.

The use of entropy as substitute to expected utility can be said to have same sort of maximization behaviour. For a hypothetical scenario, consider a game where an observer is presented with several choices and asked to pick one at random and the outcome is revealed to the ob-

TABLE 1 Model estimation results

Model	MNL		Mixed Logit		RRM (2010)		PoE		Entropy only	
Parameter	$\hat{\beta}$	t-test	$\hat{\beta}$	t-test	$\hat{\beta}$	t-test	$\hat{\beta}$	t-test	$\hat{\beta}$	t-test
ASC Bus	-2.87	-18.91	-3.00	-17.24	-2.86	-19.06	-2.84	-19.18	-2.86	-19.06
ASC Car	-1.64	-10.78	-1.62	-10.01	-1.63	-11.23	-1.63	-11.67	-1.63	-11.23
ASC Car Rental	-3.41	-14.38 -3.40	-13.79	-3.41	-14.87	-3.41	-15.38	-3.41	-14.87	
ASC Plane	-2.33	-9.28	-2.37	-8.87	-2.32	-9.92	-2.30	-10.63	-2.32	-9.92
ASC Train	-2.02	-15.27	-2.07	-13.86	-2.00	-15.46	-1.98	-15.61	-2.00	-15.46
ASC Train Hotel	0 (ref.)	-	0 (ref.)	-	0 (ref.)	-	0 (ref.)	-	0 (ref.)	-
β Cost	-0.0716	-3.25	-0.0649	-2.74	-0.186	-5.50	-0.266	-5.81	-0.186	-5.50
β Travel time	-0.578	-5.23	-0.641	-5.13	-0.0232	-3.61	-0.035	-4.03	-0.0232	-3.61
β Reliability	0.316**	0.66	0.945**	0.24	0.108**	0.67	0.167**	0.67	0.108**	0.67
σ Cost			0	-						
σ Travel time			-0.726	-3.25						
σ Reliability			-3.27**	-1.91						
Log likelihood	-2079.223		-2077.265		-2078.872		-2078.597		-2078.872	
AIC	4174.446		4176.530		4173.745		4173.195		4173.745	
BIC	4218.357		4236.907		4217.656		4217.106		4217.656	

** Not significant at 95%

server. The characteristics of the choices are not revealed and thus the observer has to decide based solely on his intuition and perception. A utility only model may not be suitable since no variables are known and the choice outcomes are usually uniformly distributed probabilities. This representation acknowledges the existence of uncertainties but often replaced with a single error term. This give rise to irrational behaviour: (1) An observer may use his perceptions on past choice attempts to select a choice. (2) An observer may choose a certain choice given the location and interaction with other alternatives. Through Bayesian inference, an entropy model can be estimated given enough attempts at the game problem which may turn out to be a non-uniform probability distribution.

CONCLUSION

The problem of irrational behaviour has been a central theme for much of behaviour economics and transportation, which contradict the ‘rational’ assumptions of utility-based models. Much like in Prospect Theory *(7)*, individuals perceive their personal benefits by distortion in payoff and adjustment of probabilities. As the correlation in the exogenous variables increases, the decision maker then focuses his or her behaviour and attention on the prior beliefs.

With the incorporation of entropy as a function of exogenous attributes in RUM, it may limit the ability for the model to discover ways at which behaviour is implied, but not described. A fundamental way of dealing with this is to use latent variables to represent the unknown aspects of heterogeneity. One interesting idea is the application of generative machine learning algorithms that has the same basis as a Boltzmann distribution *(15)*. Applying the same Bayesian inference concept gives us the ability to infer outcomes by estimation the underlying causal model.

It has been shown in *(16)* that the entropy maximizing approach is identical to the behavioural demand modelling method that follows *(17)*. According to *(16)*, non-linear constraints reflect higher momentum properties of the distribution which are realistic in nature but result in more complex formulation.

In this paper, we develop a generalized PoE framework for multinomial discrete choice models. The PoE framework provides a method for incorporating the notion of irrational behaviour into standard RUM models. The PoE probability distribution is formulated by an energy based function derived from thermodynamics. The case study provides an example on a typical mode choice model with several attributes associated with the alternatives. The results show that reformulation into a entropy based model allows for behaviour to be represented more accurately, based on significant model parameters and higher model likelihood.

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